Explaining ignorance inferences and roundness effects of modified numerals

This paper provides a unified account for three puzzles concerning modified numerals (e.g., *more than n*). [A] Superlative modifiers (*at least/at most*) lead to much stronger ignorance inferences than comparative modifiers (*more than/less than*) (e.g., Geurts and Nouwen, 2007; Kennedy, 2015; Ciardelli *et al.*, 2016). [B] It has also been argued that the distribution of modified numerals depends on the question under discussion (QUD); in particular, *more than n* is rarely used in response to a fine-grained *how many* QUD (Westera and Brasoveanu, 2014; Enguehard, 2018). [C] Finally, the roundness and contextual salience of the numeral involved plays an important role in determining the acceptability of comparative (though not superlative) modified numerals (Cummins *et al.*, 2012; Westera and Brasoveanu, 2014; Enguehard, 2018), as illustrated in (1):

(1) Mary can drink, she's at least 27/*more than 27 years old. [*legal drinking age: 21*]

Existing accounts: Cummins (2011) offers an Optimality Theory (OT) account of [C], but it wrongly predicts that *at least* should be just as sensitive to roundness as *more than*. [A] is addressed by separately stipulating that *at least* triggers stronger ignorance inferences, and [B] is not addressed in detail.

Westera and Brasoveanu (2014) offer an account of [A], based on the assumption that *at least* is typically used to address precise *how many* QUDs, while *more than* is typically used in contexts where the exact quantity does not matter (observation [B]). They justify this assumption with a corpus study showing that *more than* is used much more often with round numerals than with non-round numerals, while *at least* does not show such a strong preference. However, this corpus finding, which is plausibly closely related to the effects of roundness on acceptability (observation [C] above), remains unexplained.

Enguehard (2018) partly addresses [B] and [C], proposing an account of the roundness sensitivity of *more than* based on mandatory irrelevance implicatures. In short, the idea is that *more than n* triggers the inference that numerals directly above *n* are irrelevant, hence the incompatibility with precise *how many* QUDs and the preference for a salient or round *n*. This, however, does not capture the contrast between *more than* and *at least*.

Finally, Ciardelli *et al.* (2016) address [A], proposing that *at least n*, unlike *more than n* is inquisitive, which induces an ignorance inference. They do not address [B] or [C].

Our proposal: Building on Cummins (2011), we offer a new OT account which derives [A-C] from an ranked set of general pragmatic constraints. Given a triplet $\langle\varphi, s, Q\rangle$ consisting of an expression φ, a speaker's information state $s \subseteq W$, and a QUD Q (a partition of W), we assume the following constraints. Quality (QUAL) requires that s supports φ (i.e. $s \subseteq [\![\varphi]\!]$). Quantity (QUANT) requires that φ resolves the QUD just as well as s (i.e. s should not exclude more Q-cells than φ). Numeral salience (NSAL), a markedness constraint adapted from Cummins (2011), is violated if φ contains a numeral that is neither round nor contextually salient. Internal salience (ISAL), a new faithfulness constraint, is violated if φ contains a numeral that is not internally salient to the speaker in the sense that it does not match a boundary of the range of values that the speaker considers possible. For instance, if a speaker believes that between 6 and 10 students left, then the expressions *n / at least n / more than n students left* satisfy ISAL just in case n is 6 or 10. Finally, Complexity (COMPL) penalizes complex expressions. In line with Cummins (2011) and the processing literature on modified numerals (e.g., Cummins and Katsos, 2010), we assume that *at least* incurs two violations, *more than* incurs one, and bare numerals none. We assume the following ranking of the constraints:

(2) QUAL ≫ QUANT ≫ NSAL ≈ ISAL ≫ COMPL

Following Boersma (1997), among others, we interpret ≈ in a probabilistic manner: if the constraints NSAL and ISAL are in conflict, they do not cancel each other but either of them could take precedence at evaluation time. Finally, we assume the usual naive semantics for modified numerals (*at least n P Q* is interpreted as $|P \cap Q| \geq n$, *more than n P Q* as $|P \cap Q| > n$) and an exact semantics for numerals (the account can also easily accommodate an ambiguity theory for bare numerals).

Predictions: The two OT tableaux in Fig. 1 summarize the predictions that our account makes for production in case of fine-grained *how many* questions and polar questions, respectively. Coarse-grained *how many* questions behave like polar questions for our purposes. Since these are tableaux for production, s and Q are fixed and only expressions φ are compared and evaluated against each other. Note that the constraint QUANT has an effect with *how many* questions but not with polar questions because in the latter case, all the candidate expressions completely answer the QUD.

Candidates other than *at least* always win when the speaker has precise knowledge (when $s = \{m\}$ or $\{k\}$, as in the top two rows); *at least* only wins in ignorance contexts. On the other hand, *more than* can be optimal in the context of a polar question (or a coarse-grained *how many* question) when the speaker has exact knowledge of a non-round number ($s = \{k\}$). This contrast derives observation [A]. We also derive that when the speaker uses *at least n*, she must consider n a possible value; otherwise, ISAL would be violated and *at least n* would be harmonically bounded by *more than n*. This effect had to be stipulated in previous accounts (e.g., Geurts and Nouwen 2007; Cummins 2011).

We predict that *more than n* can be optimal in response to a polar question (or a coarse-grained *how many* question), but is hardly ever optimal with precise *how many* questions. The only exception is when n is round and s amounts to $[n+1, \ldots)$, in which case *more than n* ties with *at least* $n+1$ (to see this consider the last block in the tableau for *how many* questions, and take k to be $m+1$; then *more than m* does not incur a QUANT violation). On the other hand, *at least n* can be used with any QUD, independently of the roundness of n. This captures [B]. As a consequence, we also predict that when the QUD is underspecified, *more than* strongly suggests a coarse-grained or polar QUD, whereas *at least* does not show such a preference, as suggested in Westera and Brasoveanu (2014).

Because *more than* cannot satisfy QUAL and ISAL at the same time, it must minimally satisfy NSAL in order to be optimal. In contrast, *at least* can satisfy QUAL and ISAL together. Because of this, *more than* is predicted to be used with round or salient numerals irrespective of whether the speaker has exact knowledge or not. This captures [C], including the non-sensitivity of *at least* to roundness, which was left unexplained by previous accounts (Cummins, 2011; Enguehard, 2018).

Flipping the OT tableaux yields predictions about comprehension rather than production, which align with the experimental findings of Westera and Brasoveanu (2014): Ignorance inferences depend entirely on the QUD, and not on the choice of modified numeral. Thus, a small set of general pragmatic constraints can capture a wide range of empirical findings on modified numerals that have heretofore eluded a unified analysis, without requiring any ad-hoc semantic assumptions.

Precise *how many* questions

s	φ	QUAL	QUANT	ISAL	NSAL	COMPL
$\{m\}$	m					
	mt m-1		*!	*	*	*
	al m		*!			**
$\{k\}$	k				*	
	mt k-1		*!	*	*	*
	mt m		*!	*		*
	al k		*!		*	**
	al m		*!	*		**
$[m,\ldots)$	m	*!				
	mt m-1			*	*	*
	al m					**
$[k,\ldots)$	k	*!			*	
	mt k-1			*!	*	*
	mt m		*!	*		*
	al k				*	**
	al m		*!	*		**

Polar questions $[m,\ldots)$?

s	φ	QUAL	QUANT	ISAL	NSAL	COMPL
$\{m\}$	m					
	mt m-1			*!	*!	*
	al m					**!
$\{k\}$	k				*	
	mt k-1			*!	*!	*
	mt m			*		*
	al k				*	**!
	al m			*		**!
$[m,\ldots)$	m	*!				
	mt m-1			*	*	*
	al m					**
$[k,\ldots)$	k	*!			*	
	mt k-1			*!	*!	*
	mt m			*		*
	al k				*	**
	al m			*		**!

Fig. 1: OT tableaux for *how many* and polar questions and four cases of speaker's information state. m is a round or contextually salient number, k a non-round number greater than m, *mt* = *more than*, *al* = *at least*. Winning candidate(s) are marked in green. We do not show losing candidates that are not relevant for our discussion here.

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