Independence in distributivity

Introduction Distributivity as expressed in (1) is ambiguous as to whether the witness of the indefinite *a book* may (1a) or may not (1b) depend on (i.e., may co-vary with) the so-called distributivity key (Key), i.e., the plural referent denoted by *the students*.

(1) The students each read a book.

a. Dependent: $\forall x[\textbf{student}\ x](\exists y[\textbf{book}\ y](\textbf{read}\ y\ x))$

b. Independent: $\exists y[\textbf{book}\ y](\forall x[\textbf{student}\ x](\textbf{read}\ y\ x))$

Recent studies identified a host of markers that force indefinites to participate in the dependent interpretation. Notable examples are numeral reduplication in various languages (Henderson 2014, a.o.) and English binominal *each* (Milačić et al. 2015, Champollion 2015, Kuhn 2017). This paper identifies a novel typology of distributive quantification, one that signals independence. The empirical evidence comes from Cantonese, which uses a verbal suffix *saai* as a strategy to mark distributivity. Below, I first discuss how different types of indefinites in *saai*-distributivity seem to oscillate between dependence and independence. Then I propose an analysis, couched in Dynamic Plural Logic (DPlL, van den Berg 1996, Nouwen 2003), that takes *independence* as the default, but derives dependence in a principled way.

(In)dependence *Saai* is typically analyzed as a distributivity marker in Cantonese (Lee 1994; Tang 1996; Lee 2012), based on the observation that a sentence like (2) is verified by each student buying one or more books but not a group purchase. The distributivity reading here is unexceptional as the witness associated with the bare noun object is ambiguous between the dependent and the independent interpretation.

(2) Di-hoksaang$_{\text{Key}}$ maai-**saai** syu.
the.PL-student buy-SAAI book(s)
'The students all bought (one or more) books'

Interestingly, indefinites other than bare nouns in *saai*-distributivity only exhibit the independent interpretation (3). Similarly, a disjunction introduced in *saai*-distributivity also lacks witness variation (4). In other words, indefinites and disjunction behave as if they are outside the scope of distributive quantification.

(3) Di-hoksaang maai-saai jat-bun syu.
the.PL-student buy-SAAI one-CL book
'They all bought a book, the same book.'

(4) Di-hoksaang maai-saai Emma waatze Jane Eyre.
the.PL-student buy-*saai* Emma or Jane Eyre
'They all bought Emma or they all bought Jane Eyre.'

However, indefinites with a pronoun bound by the Key may exhibit dependence (5).

(5) Di-hoksaang^x maai-saai **zigei**$_x$ zungji-ge jat-bun syu.
the.PL-student buy-SAAI self like-MOD one-CL book
'They all bought a book that they like.'

It seems hard to make sense of the above oscillation between dependence and independence. For one thing, if dependence and independence are to be captured by scope, on what basis do we decide where to scope an indefinite (or disjunction) relative to *saai*?

Proposal I propose that *saai* as a verbal affix has a **narrow** distributive scope—it only scopes over the verb (see also Tang 1996). Consequently, all the other constituents are interpreted outside the scope of distributivity, giving rise to scope independence of non-verbal elements.

The proposal is couched in DPlL (van den Berg 1996, Nouwen 2003), in which sentences denote relations between sets of variable assignments, known as info-states. Variables are introduced into an info-state by random assignment (6), which does not bring about dependency between the variable being introduced and extant variables (Figure 1, contra Brasoveanu 2008). Lexical relations are evaluated collectively (7). A distributivity operator, defined in (8), splits an info-state along the Key dimension (i.e., the values stored in the subscripted variable) into different sub-info-states. It allows a predicate to be collectively evaluated relative to each sub-info-state, bringing about distributivity. Importantly, when random assignment, i.e., $\exists x$, occurs inside the scope of the distributivity operator, independence is restricted to each sub-info-state. As a result, dependency may occur after distributivity is evaluated (see Figure 2).

(6) $G[\![\exists x]\!]H = \mathbb{T}$ iff $\exists d.H = \{G^{x \to d} | g \in G \,\&\, d \in D\}$

(7) $G[\![R x_n, ..., x_1]\!]H = \mathbb{T}$ iff $G = H$ and $\langle G\, x_1, ..., G\, x_n \rangle \in \mathcal{I}(R)$

(8) $G[\![\delta_x(\phi)]\!]H = \mathbb{T}$ iff $G\, x = H\, x$ and $\forall a \in G\, x. G|_{x=a}\ [\![\phi]\!]\ H|_{x=a} = \mathbb{T}$

Saai, as defined in (9), combines with verbs and introduces distributive quantification scoping over only the verbal relation ($\vec{y}$ stands for any sequence of variables, including none). Compositionally, all other constituents are outside the distributive operator δ_x. Given the lexical entries in (10) and (11), the denotation of (3) is derived as in (12). Because the (underlined) random assignments associated with the indefinite (and the disjunction) are introduced outside the scope of the distributivity operator, it is correctly predicted that they do not co-vary with values in the Key.

(9) $\textbf{saai} := \lambda R \lambda \vec{y} \lambda x. \delta_x(R\ \vec{y}\ x)$

(10) $\textbf{one book} := \lambda P. \exists x \wedge \textbf{book}\ x \wedge |x| = 1 \wedge P\ x$

(11) a. $\textbf{the students} := \lambda P. \exists x \wedge \textbf{max}_x(\textbf{student}\ x) \wedge P\ x$

b. $G[\![\textbf{max}_x(\phi)]\!]H = \mathbb{T}$ iff $G[\![\phi]\!]H = \mathbb{T}$ and $\neg \exists H'. H' \supset H \,\&\, G[\![[x]\ \wedge\ \phi]\!]H'$

(12) $\exists x \wedge \textbf{max}_x(\textbf{student}\ x) \wedge \underline{\exists y} \wedge\ \textbf{book}\ y \wedge |y| = 1 \wedge \delta_x(\textbf{read}\ y\ x)$

Bound pronouns Although indefinites are outside the scope of the distributive quantification introduced by *saai*, they may co-vary with the Key if they contain a pronoun bound by the Key (5). Specifically, an indefinite with a bound pronoun is allowed to introduce a set of values dependent on the binder of the bound pronoun, as done in (13) and further spelled out in (14) (see van den Berg 1996). Because distributivity is dynamic in DPlL, the dependency between the students and the books is stored and fed to the distributivity induced by *saai*. As a result, the logical form in (13) correctly represents the interpretation of (5)—each student bought a book that they like.

(13) $\exists x \wedge \textbf{max}_x(\textbf{student}\ x) \wedge \underline{\exists y_{Rx}} \wedge \textbf{book}\ y \wedge |y| = 1 \wedge \textbf{like}\ y\ x \wedge \delta_x(\textbf{read}\ y\ x)$

(14) $\exists y_{Rx} := \delta_x(\exists y \wedge Ryx)$, where R is a contextually relevant relation between x and y.

An account couched in first order logic either requires the indefinite with a bound pronoun to fall inside the scope of *saai*-distributivity, bleeding the independence generalization, or predicts an overly strong reading: there is a book collectively liked by the students and each student read it.

Bare nouns Recall that scope independence also does not hold for indefinites in the form of bare nouns (2). I propose that a bare noun may indeed introduce a discourse variable outside the scope of *saai*, but the variable stores a **kind value**, akin to that of a proper name, as shown in (15) (see Carlson 1977 and Dayal 1999 for evidence that discourse anaphora to kinds is necessary). Witness variation comes in when Derived Kind Predication (DKP, Carlson 1977 and Chierchia 1998), a sortal repair strategy is applied inside the scope of distributivity. DKP is defined in terms of DPlL here in (16) (where **inst** is an instantiation relation between a kind and a member of the kind). Since the random assignment introduced by DKP is inside the scope of distributivity, witness variation is allowed (see also Champollion et al. 2017 for a related idea).

(15) $\exists x \wedge \textbf{max}_x(\textbf{students}\ x) \wedge \exists y \wedge y = \textbf{book-kind}\ \wedge \delta_x(\textbf{DKP}(\textbf{read})\ y\ x)$

(16) $\textbf{DKP} := \lambda R \lambda y \lambda x. \exists z\ \wedge \textbf{inst}\ zy\ \wedge\ Rzx$

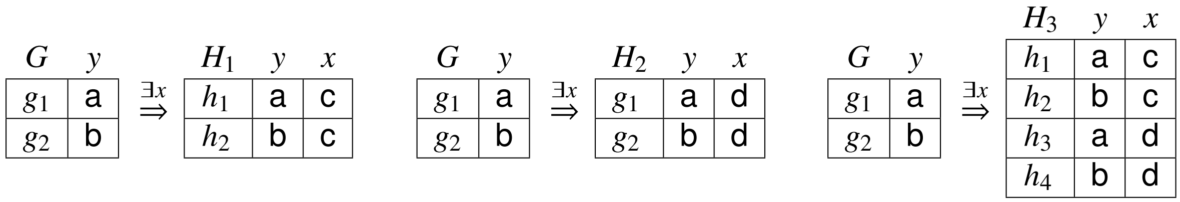

Figure 1: Dependence-free random assignment

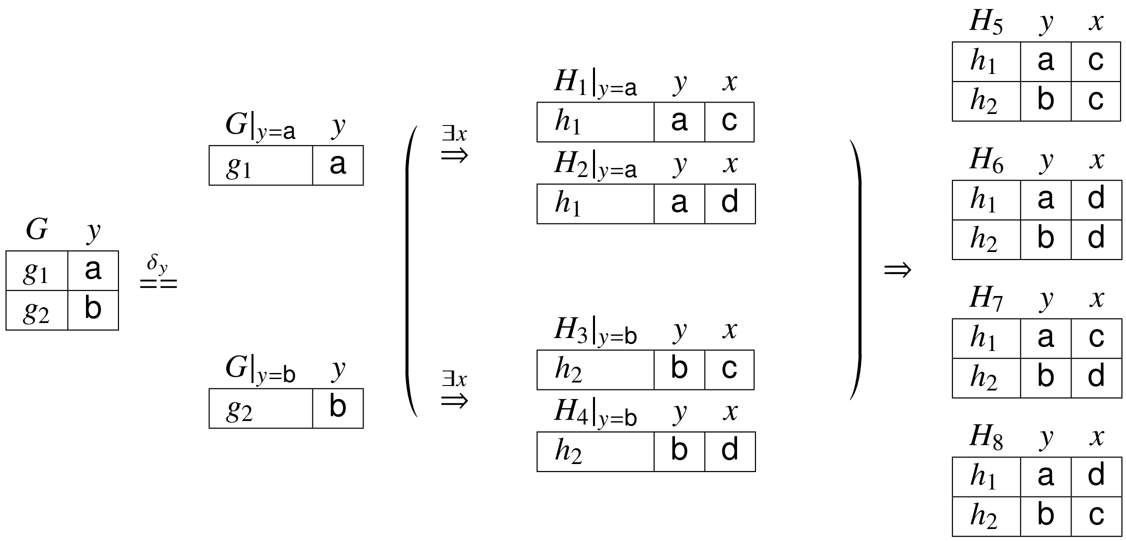

Figure 2: Evaluating random assignment inside the scope of distributivity (δ_x) may give rise to dependence. In the final output, H_7 and H_8 exhibit dependence between x and y.

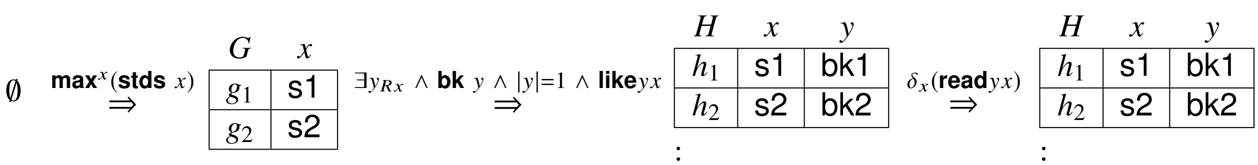

Figure 3: A bound pronoun allows the introduction of dependence

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