Overt and covert proportional partitives

Overview I analyze partitive and partitive-like constructions with proportional modifiers such as 70%. This includes overt partitives (70% *of the students*) and two constructions that use silent partitive structures: one adjectival (70% *female*) and one nominal (70% *women*).

The base case: 70% *full/#tall* The analysis of proportional partitives will be built on a degree-based approach to proportion phrases as required for cases like 70% *full*: as seen in (1), [70%] takes an adjective denotation A (a relation between degrees and entities) and entity x, returning true iff the maximal degree to which x is A is at least 70% of absolute maximal A-ness. This definition accounts for Kennedy & McNally's (2005) observation that proportional modifiers require closed scales (cf. #70% *tall*), since [70%] requires that A's scale have a maximal and minimal degree.

Overt proportional partitives: 70% of the students Next we move on to partitives like 70% of the students. As shown in (2), the crucial work is done by [[of]], which takes an entity x (the referent of the students) and returns an adjective-like denotation with contextually-determined measure function μ^c , with two restrictions: the (second) entity y must be a part of x, and the degree must not exceed the measurement of x by μ^c . (As noted by Krifka (1989) for pseudopartitives, an additional constraint on μ^c must apply, allowing 12 ounces of gold (weight) but not #12 carats of gold (purity); this constraint also applies for partitives. However, it is irrelevant for our purposes, so I exclude it.) All of our examples will use the cardinality measure function, so $\mu^c(\alpha)$ will be replaced with $|\alpha|$.

When $[\![of]\!]$ is applied to $[\![the students]\!]$ —the latter being $\sigma x[students(x)]$, the maximal plurality of students—the result is (3). (3) is defined for entities that are student pluralities (including atoms), and for degrees not exceeding the cardinality of the plurality of all students. Thus, the minimum degree is 0 (the minimal cardinality), and the maximum degree is $|\sigma x[students(x)]|$. As a result, $[\![70\%]]([\![of the students]])$ is as in (4), true of a plurality of students iff its cardinality is at least 70% of the cardinality of the sum of all the students. This then combines with the silent determiner SOME, whose denotation is the standard $\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle$ -type existential quantifier. The result is an $\langle \langle e, t \rangle, t \rangle$ -type quantifier true of predicate Q iff Q holds of 70% of the students, as desired.

Covert proportional partitives I: 70% *female* The first construction using a covert proportional partitive is of the sort represented by 70% *female* in (6). Note that 70% is not a floated quantifier akin to *all*: (7) shows that 70% cannot appear in other places that floated quantifiers can, and (8) that 70% *female* can be used attributively. Thus, it appears that 70% (indirectly) modifies *female*, not *the students*. To illustrate how this can be done, consider the following paraphrase of (6): *The students are a plural individual* 70% *of which is female*. In this paraphrase, the relative pronoun *which* serves as the (first) argument of *of*. I adopt a similar analysis for (6): a relative pronoun *Op* starts as the complement of a silent OF, undergoing short wh-movement and triggering lambda abstraction over this argument of OF; the proposed syntax is as in (9), with the semantic result in (10), true of an individual *x* iff there is a part *y* of *x* that is female, and whose cardinality is at least 70% of *x*'s. (The 1 after *Op* is Heim & Kratzer's (1998) trace-binding node, responsible for lambda abstraction.) Combining this with [[the students]] gets the right interpretation: 70% of the students are female.

Covert proportional partitives II: 70% women Ahn & Sauerland (2015, 2017) note the seemingly non-conservative construal of proportional DPs in (11), which is true iff 70% of the hirees were

women. They also note that this construction seems to be focus-sensitive: (12), with focus on *Italian*, means not that 70% of the *hirees* were Italian women, but that 70% of the *female hirees* were Italian.

Let's begin with (11). We start with the structure in (9), replacing *female* with *women*. The denotation of this constituent is a predicate true of pluralities 70% of which are women. There are two clear candidates for how this $\langle e, t \rangle$ -type constituent composes with *hired*: either via a covert, presumably indefinite determiner (perhaps SOME), or in direct composition via Chung & Ladusaw's (2004) RESTRICT operation. But on both approaches, the predicted result is existential quantification over [70% women], resulting in truth conditions that are far too weak: namely, that there is some 70% female collection of individuals that the company hired. (To see that this is too weak, consider the fact that the plurality of all women that the company hired clearly meets this condition, even if this plurality constitutes less than 70% of the total hirees.) Thus, these two compositional paths must be closed to us. This could perhaps be justified on syntactic/interface grounds: the structure for 70% *women* might not be of the right syntactic type to combine directly with a(n indefinite) determiner, and RESTRICT appears to have a relatively narrow syntactic distribution anyway. Either way, suppose these routes are unavailable. What happens next?

I propose that 70% women undergoes QR, leading to the LF in (13). In order for this to compose properly, we must adopt two principles for trace interpretation and lambda conversion: (I) a trace must be a free variable of the right semantic type to serve as the argument of its sister; and (II) the result of lambda abstraction must be type-shifted to serve as an argument to its sister. Thus, $[t_2]$ must be of type *e*, and the result of lambda abstraction over t_2 must also be shifted to type *e* to compose with 70% women. The latter is accomplished by applying the referential σ operator to the lambda abstraction—essentially, Partee's (1987) *iota* type-shift. When this is fed to [70% women], the result is as in (14), essentially paraphrasable as *The company's hirees were 70% female*, as desired.

As for (12), this falls out from an independent observation: focus in the scope of a DP can restrict quantification to the disjunction of the focus alternatives (see Beaver & Clark 2008 and sources therein). For example, suppose that students have a choice between taking an exam and writing a paper, and the test-taking students could choose to take the exam on Monday or Tuesday. Here, (15), with focus on *on Tuesday*, has a reading in which 70% of the students *who took the exam* (on some day) took it on Tuesday. Now two things are worth noticing with respect to the relation between (15) and (12). First, on the analysis adopted here, the scope of the mostly silent partitive in (12) is *Italian women*, meaning (12), like (15), involves focus in the scope of the partitive. And second, for whatever reason the focus-effected domain restriction in (15) is realized as a restriction on the definite DP *the students*, so since *Op* stands in for the definite DP, this is where the restriction will be enforced in our account.

With this in mind, suppose that we define $[\![Op]\!]$ as in (16), where *R* is a free $\langle e, t \rangle$ -type domain restriction variable. $[\![Op]\!]$ takes a predicate *P* (the result of lambda-abstraction) and returns a predicate true of an entity *x* iff *P* is true of that part of *x* of which *R* holds. If there is no domain restriction (i.e., $R = \lambda x$. \top), then $[\![Op]\!]$ is the identity function, meaning that the result is the same as before. But in cases where there is focus in the scope of the partitive, as in 70% [Italian]_{*F*} women, *R* is (or can be) set to the disjunction of the focus alternatives of the nuclear scope ([Italian]_{*F*} women). Since the focus alternatives of [Italian]_{*F*} women are all of the form λx . $Q(x) \land$ women(x) for various Q, their grand union/disjunction, and thus *R*, will simply be λx . women(x). When this restriction is applied to $[\![Op]\!]$, the result comes out as equivalent to (17): true iff Italian female hirees make up at least 70% of the total female hirees. We thus derive the desired truth-conditions by combining our analysis with independently attested principles of focus-derived quantifier restriction.

Examples

(1)
$$\llbracket 70\% \rrbracket = \lambda A \lambda x. \frac{\max\{\{d \mid A(d)(x)\}\} - \min(\operatorname{RNG}(A))}{\max(\operatorname{RNG}(A)) - \min(\operatorname{RNG}(A))} \ge .7$$

(where $\operatorname{RNG}(A) = \{d \mid \exists x[A(d)(x) \text{ is defined}]\}$)
(2) $\llbracket of \rrbracket^c = \lambda x \lambda d \lambda y : y \subseteq x \land \mu^c(x) \ge d. \mu^c(y) \ge d$
(3) $\llbracket of \text{ the students} \rrbracket^c = \lambda d \lambda y : y \subseteq \sigma x[\text{ students}(x)] \land |\sigma x[\text{ students}(x)]| \ge d. |y| \ge d$
(4) $\llbracket 70\% \text{ of the students} \rrbracket^c = \lambda y : y \subseteq \sigma x[\text{ students}(x)]. \frac{|y|}{|\sigma x[\text{ students}(x)]|} \ge .7$
(5) $\llbracket \text{some} \rrbracket (\llbracket (4) \rrbracket) = \lambda Q. \exists y[y \subseteq \sigma x[\text{ students}(x)] \land \frac{|y|}{|\sigma x[\text{ students}(x)]|} \ge .7 \land Q(y) \rrbracket$
(6) The students are $\underline{70\% \text{ female}}.$ (7) The students (all/*70%) must (all/*70%) be female.
(8) The $\underline{70\% \text{ female cast}}$ did a fantastic job.
(9) $\llbracket \text{Op 1} \llbracket \text{ some 70\% \text{ or } t_1 \rrbracket \text{ female} \rrbracket \rrbracket$
(10) $\llbracket (9) \rrbracket = \lambda x. \exists y[y \subseteq x \land \frac{|y|}{|x|} \ge .7 \land \text{ female}(y) \rrbracket$
(11) The company hired $\underline{70\% \text{ or } t_1}$ momen.
(12) The company hired $\underline{70\%}$ [Italian]_F women.
(13) $\llbracket \text{Op 1} \llbracket \text{ some 70\% \text{ or } t_1 \rrbracket \text{ women} \rrbracket \rrbracket 2 \text{ the company hired } \frac{1}{|\sigma x[\text{ the company hired } x]} \ge .7 \land \text{ women}(y) \rrbracket$
(14) $\llbracket (13) \rrbracket = 1 \text{ iff } \exists y[y \subseteq \sigma x[\text{ the company hired } x] \land \frac{|y|}{|\sigma x[\text{ the company hired } x]} \ge .7 \land \text{ women}(y) \rrbracket$
(15) $70\% \text{ of the students took the exam [on Tuesday]_{F}}.$
(16) $\llbracket \text{Op } \rrbracket = \lambda P \lambda x. P(x \sqcap \sigma y[R(y)])$ (where $x \sqcap y = \sigma z[z \subseteq x \land z \subseteq y]$)

. . . .

(17)

 $\llbracket (12) \rrbracket = 1 \text{ iff } \exists y [y \sqsubseteq \sigma x [\text{the company hired } x \land \text{women}(x)] \land$

 $\frac{|y|}{|\sigma x[\text{the company hired } x \land \text{women}(x)]|} \ge .7 \land \text{italian}(y) \land \text{women}(y)]$

References

- Ahn, Dorothy & Uli Sauerland. 2015. The grammar of relative measurement. In Sarah D'Antonio, Mary Moroney & Carol Rose Little (eds.), Semantics and Linguistic Theory (SALT) 25, 125-142. LSA and CLC Publications.
- Ahn, Dorothy & Uli Sauerland. 2017. Measure constructions with relative measures: Towards a syntax of non-conservative construals. The Linguistic Review 34(2). 215–248.
- Beaver, David I. & Brady Z. Clark. 2008. Sense and Sensitivity: How Focus Determines Meaning. Oxford: Wiley-Blackwell.
- Chung, Sandra & William Ladusaw. 2004. Restriction and Saturation. Cambridge, MA: MIT Press.
- Heim, Irene & Angelika Kratzer. 1998. Semantics in Generative Grammar. Oxford: Blackwell.
- Kennedy, Christopher & Louise McNally. 2005. Scale structure, degree modification, and the semantics of gradable predicates. Language 81(2). 345-381.
- Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch, Johan van Benthem & Peter van Emde Boas (eds.), Semantics and Contextual Expression, 75-115. Dordrecht: Foris.
- Partee, Barbara. 1987. Noun phrase interpretation and type-shifting principles. In Jeroen Groenendijk, Dick de Jongh & Martin Stokhof (eds.), Studies in discourse representation theory and the theory of generalized quantifiers, 115-141. The Netherlands: Foris.