Counterfactuals and indeterminacy

0. Overview. Classical semantics for counterfactuals (Stalnaker 1968, Lewis 1973) offer a forced choice between Conditional Excluded Middle and Duality (which here I construe as the claim that $\phi \Box\!\!\rightarrow \neg\psi$ and $\phi \Diamond\!\!\rightarrow \psi$ are incompatible). Yet both principles seem valid in natural language. I suggest that the puzzle is a conditional generalization of Yalcin's (2007) epistemic contradictions puzzle, and develop an expressivist/dynamic semantics that vindicates both principles. The key move is to introduce a notion of indeterminacy in the semantics for counterfactual modality.

1. WMC and CEM. Natural language counterfactuals seem to satisfy two logical principles:

Would-Might Contradiction. (WMC) $(\phi \Box\!\!\rightarrow \neg\psi) \wedge (\phi \Diamond\!\!\rightarrow \psi) \vDash \bot$

Conditional Excluded Middle. (CEM) $\vDash (\phi \Box\!\!\rightarrow \psi) \vee (\phi \Box\!\!\rightarrow \neg\psi)$

The argument for WMC is that conjunctions of the relevant counterfactuals are contradictory, as in (1). The argument for CEM is that counterfactuals are scopeless with respect to negation, as illustrated by the equivalence of (2)-a and (2)-b. (See also Higginbotham 1986, von Fintel 1997.)

(1) # If Maria had passed, Frida wouldn't have; but, if Maria had passed, Frida might have.

(2) a. It's not the case that, if Frida had taken the exam, she would have passed.
b. If Frida had taken the exam, she would not have passed.

Unfortunately, WMC and CEM classically entail the absurd equivalence of $\phi \Diamond\!\!\rightarrow \psi$ and $\phi \Box\!\!\rightarrow \psi$.

Homogeneity? Some theorists (von Fintel 1997, Schlenker 2004, Križ 2015 a.o.) vindicate CEM by treating counterfactuals as universal quantifiers with a homogeneity requirement, similarly to plural definites. I lack space to discuss these views in detail, but let me point to a disanalogy between counterfactuals and homogeneity triggers: probability judgments work very differently for the two (see Cremers et al. 2017). This encourages the exploration of alternatives.

2. Analogy with epistemic contradictions. Yalcin (2007; see also Veltman 1985) points out that conjunctions like (3) (which he dubs 'epistemic contradictions') sound inconsistent.

(3) It's not raining and it might be raining.

Yet the inconsistency of $\neg\phi$ and $\Diamond\phi$ classically entails the equivalence of $\Diamond\phi$ and ϕ, which is absurd. Yalcin's puzzle is generated by two classically incompatible principles (below, left); the puzzle in §1 by three classically incompatible principles (below, right). The analogy is glaring and encourages exploring the prospect for an expressivist/dynamic semantics for counterfactuals.

		CEM	$\vDash (\phi \Box\!\!\rightarrow \psi) \vee (\phi \Box\!\!\rightarrow \neg\psi)$
Epistemic Contradiction.	$\neg\phi \wedge \Diamond\phi \vDash \bot$	**WMC**	$(\phi \Box\!\!\rightarrow \neg\psi) \wedge (\phi \Diamond\!\!\rightarrow \psi) \vDash \bot$
Nonfactivity.	$\Diamond\phi \nvDash \phi$	**Noncollapse. (NC)**	$\phi \Diamond\!\!\rightarrow \psi \nvDash \phi \Box\!\!\rightarrow \psi$

3. Indeterminacy. Following Lewis (1973), I use a relation $\preceq_w$ of comparative closeness between worlds; $\preceq_w$ is transitive and strongly connected and induces a total preorder on worlds. I depart from Lewis in one respect: ties in the preorder represent indeterminacy, whether in facts or in 'counterfacts'. (For the notion of a 'counterfact', see Hawthorne 2005, Moss 2013 a.o.) For illustration, consider:

Coin flip. Frida will flip a coin tomorrow. Coin flips are indeterministic. Hence it is indeterminate whether a heads-world or a tails world (i.e. w_H and w_T; see Fig. 1) is actual.

Missed coin flip. Yesterday Frida didn't flip the coin, but it was a historically open possibility that she might have flipped it. Hence there is currently only one candidate for what world is actual, but two candidates for what world would be actual (w_H and w_T), had she flipped (see Fig. 2). Hence it is indeterminate which world is actualized on the supposition that Frida flipped the coin.

4. States and paths. The semantics crucially exploits two new notions.

A **state** is a set of worlds s determined by all the facts that are settled at a given time. E.g., in the first coin flip example (Fig. 1) the actual state at the relevant time is $\{w_{\text{H}}, w_{\text{T}}\}$.

A **path** is a linear ordering of worlds in the domain that refines a Lewisian preorder (see Figure 3).

- A path represents a fully determinate scenario, in which both facts and counterfacts are settled.
- A path can be represented either as a sequence of worlds, or as a sequence of larger and larger sets of worlds. I choose the latter representation.

A relation linking states and paths: p **passes through** s iff s is a member of p.
Intuitively, p passes through s iff p is an admissible precisification of what things are like in s.

Example. In the Fig. 2 scenario there are two paths, both passing through the actual state $\{w_{\bar{\text{F}}}\}$:

$$\{\{w_{\bar{\text{F}}}\}, \{w_{\bar{\text{F}}}, w_{\text{T}}\}, \{w_{\bar{\text{F}}}, w_{\text{H}}, w_{\text{T}}\}\} \qquad \{\{w_{\bar{\text{F}}}\}, \{w_{\bar{\text{F}}}, w_{\text{H}}\}, \{w_{\bar{\text{F}}}, w_{\text{H}}, w_{\text{T}}\}\}$$

5. Formal semantics I relativize intepretation to three parameters: a **path** p; a **preorder function** g, i.e. a function from a state to a preorder; a **state** s. I assume a background model $\langle W, V\rangle$, with W a set of worlds and V a valuation function from atomic sentences and worlds to $\{0, 1\}$.

Nomodal fragment. The clauses for atomic sentences and connectives are as follows:

Atoms: $[\![A]\!]^{p,g,s} = 1$ iff $w : \mathsf{min}(p) = \{w\}$, is s.t. $V(w, A) = 1$
where $\mathsf{min}(p)$ is the smallest non-empty member of p, if there is one; $\{\lambda\}$, otherwise.
$[\![\neg\phi]\!]^{p,g,s} = 1$ iff $[\![\phi]\!]^{p,g,s} = 0$
$[\![\phi \vee \psi]\!]^{p,g,s} = 1$ iff $[\![\phi]\!]^{p,g,s} = 1$ or $[\![\psi]\!]^{p,g,s} = 1$

Counterfactuals. We first define the notion of an update of a path with respect to a formula, and then define entries for counterfactuals using the latter.

Update of p with respect to ϕ: $p[\phi] = p \Cap (\bigcup P)[\phi]$
(with: $P \Cap i = \{p_1 \cap i, \ldots, p_n \cap i \ldots\}$)
$[\![\phi \mathrel{\Box\!\!\rightarrow} \psi]\!]^{p,g,s} = 1$ iff $[\![\psi]\!]^{p[\phi],g,s} = 1$
$[\![\phi \mathrel{\Diamond\!\!\rightarrow} \psi]\!]^{p,g,s} = 1$ iff $\exists p' \in s$ s.t. $[\![\psi]\!]^{p'[\phi],g,s} = 1$

6. Consequence. The system allows for several notions of consequence. The one that matters for our purposes is preservation of a notion of determinate truth, or truth at a state. Definitions are below:

Truth at a state.
ϕ is true at s (relative to g) iff $\forall p \in g$ s.t. p passes through s, $[\![\phi]\!]^{p,g,s} = 1$

State consequence.
$\phi_1, \ldots, \phi_n \vDash_S \psi$ iff, for all s such that $\phi_1, \ldots, \phi_n$ are true at s, ψ is true at s.

Crucially, we have that state consequence validates all of CEM, WMC, and NMC..

Fact. $\vDash_S (\phi \mathrel{\Box\!\!\rightarrow} \psi) \vee (\phi \mathrel{\Box\!\!\rightarrow} \neg\psi)$ $\qquad (\phi \mathrel{\Box\!\!\rightarrow} \neg\psi) \wedge (\phi \mathrel{\Diamond\!\!\rightarrow} \psi) \vDash_S \bot$ $\qquad \phi \mathrel{\Diamond\!\!\rightarrow} \psi \nvDash_S \phi \mathrel{\Box\!\!\rightarrow} \psi$

7. Conclusion. We can vindicate two plausible but classically incompatible principles of counterfactual logic by using a semantics that involves a notion of indeterminacy. The obvious formal analogies with non-truth-conditional semantics for epistemic modality (in particular data semantics, see Veltman 1985) suggest that the framework might be generalized to other modal flavors.

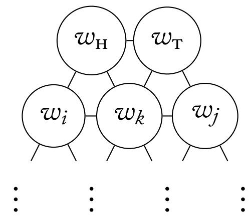


Figure 1: indeterminacy about facts (it's indeterminate whether w_{H} or w_{T} is actual).

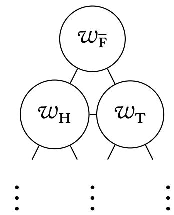


Figure 2: indeterminacy about counterfacts (it's indeterminate whether w_{T}or w_{H}would have been the case, had the coin been flipped)

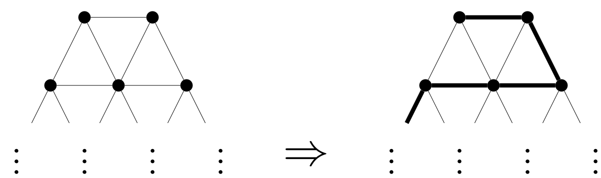
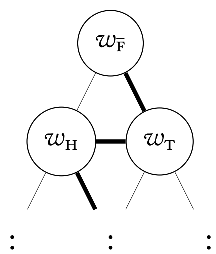


Figure 3: extracting a path from a Lewisian preorder.



	TRUE AT PATH	TRUE AT STATE
Frida didn't flip the coin.	✓	✓
Frida flipped or Frida didn't flip.	✓	✓
If Frida had flipped, it would have landed tails.	✓	✗
If Frida had flipped, it might have landed heads.	—	✓
(If F had flipped, tails) or (If F had flipped, heads.)	✓	✓

Figure 4: a sample path-state combination with some sample predictions.

References. Cremers, Križ, and Chemla 2017, "Probability Judgments of Gappy Sentences" · von Fintel 1997, "Bare Plurals, Bare Conditionals, and *Only*" · Kratzer 2012 *Modals and Conditionals: New and Revised Perspectives* · Hawthorne 2005, "Chance and Counterfactuals" · Križ 2015, *Aspects of homogeneity in the semantics of natural language* · Moss 2013,"Subjunctive Credences and Semantic Humility" · Schlenker 2004 "Conditionals as Definite Descriptions: A Referential Analysis" · Stalnaker 1968, "A Theory of Conditionals" · Veltman 1985, *Logics for Conditionals* · Veltman 1996, "Defaults in Update Semantics" · Yalcin 2007, "Epistemic modals"